Exercises Chapter IV

Mathematical Methods of Bioengineering

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Contents

This is a guide to the exercises that you can solve. If you fall short, there are more similar exercises in the books of the subject. You can ask me questions at the end of class or in tutoring.

The idea is that you solve the exercises and do them on the blackboard in class. Each time you go to the board, it will count to the 5% of the final mark. You must participate at least 3 times in order to get the full 5% and at least 6 times to raise the final grade by +0.5 points.

1 Vectors

2 Differentiation in Several Variables

3 Vector Valued Functions

4 Maxima and Minima in Several Variables

4.2 Extrema of functions

1. Identify and determine the nature of the critical points of the given functions in $\mathbb{R}^3$.

   (a) $f(x, y) = 2xy - 2x^2 - 5y^2 + 4y - 3$
   (b) $f(x, y) = \ln(x^2 + y^2 + 1)$
   (c) $f(x, y) = e^x \sin y$
   (d) $f(x, y) = e^{-y}(x^2 - y^2)$
   (e) $f(x, y) = \cos x \sin y$

2. Identify and determine the nature of the critical points of the given functions in $\mathbb{R}^4$.

(a) $f(x, y, z) = xy + xz + 2yz + \frac{1}{x}$

(b) $f(x, y, z) = e^x(x^2 - y^2 - 2z^2)$

3. Find all critical points of $f(x, y) = \dfrac{2y^3 - 3y^2 - 36y + 2}{1 + 3x^2}$ and identify, if any, all extrema of f.

4. Under what conditions on the constant k will the function $f(x, y) = kx^2 - 2xy + ky^2$ have a nondegenerate local minimum at $(0, 0)$. What about a local maximum?

5. (a) Consider the function $f(x, y) = ax^2 + by^2$, where a and b are nonzero constants. Show that the origin is the only critical point of f, and determine the nature of that critical point in terms of a and b.

   (b) Do the same for $f(x_1, x_2, ..., x_n) = a_1x_1^2 + a_2x_2^2 + ... + a_nx_n^2$ where a_i, $i = 1, 2, ..., n$ is a nonzero constant.

6. Suppose that you are in charge of manufacturing two types of television sets. The revenue function, in dollars, is given by

$$R(x, y) = 8x + 6y - x^2 - 2y^2 + 2xy$$

   where x denotes the quantity of model X sets sold, and y the quantity of model Y sets sold, both in units of 100. Determine the quantity of each type of set that you should produce in order to maximise the resulting revenue.

7. Find the points on the surface $xy + z^2 = 4$ that are closest to the origin. (Hint: minimise the square of the distance to the origin.)

8. Show that the largest rectangular box (in the sense of volume) having a fixed surface area must be a cube.